3rd Japanese–German Number Theory Workshop

Monday 20th – Friday 24th November, 2017
Max Planck Institute, Lecture Hall
Organizers: H. Aoki, J. Kaszian, P. Moree, K. Tasaka

Abstracts

Claudia Alfes-Neumann (Univ. Paderborn)
Regularized theta lifts of integral weight harmonic Maass forms I

In these two talks we will give an overview on developments in the theory of regularized theta lifts of harmonic Maass forms of integral weight. In particular, we report on recent work which extends the Shintani theta lift to harmonic Maass forms. This yields interesting number theoretic applications.

Hiroki Aoki (Tokyo Univ. of Science), ABKLS talk
On the structure of mixed weight Hilbert modular forms

In this talk we discuss joint work with Sho Takemori on Hilbert modular forms over the real quadratic field of discriminant 5, with respect to its full modular group. The graded ring of all Hilbert modular forms of parallel weight was determined by Gundlach. By using his result and some elementary technique, we establish a structure theorem on mixed weight Hilbert modular forms.

Hiraku Atobe (Univ. of Tokyo)
On the Miyawaki lifts of hermitian modular forms

We construct a lifting from a elliptic modular cusp form f and a hermitian modular cusp form g of degree r to a hermitian modular cusp form of degree $n+r$. This lift is the so-called Miyawaki lift. When f and g are "Hecke eigenforms", its lift is also a "Hecke eigenform" (if it is not identically zero). Moreover, we determine explicitly the standard L-function and A-parameter associated to the Miyawaki lift.

Minoru Hirose (Kyushu Univ.)
On a certain class of linear relations among the multiple zeta values arising from the theory of iterated integrals

In this talk, we consider iterated integrals on a projective line minus generic four points and introduce a new class of linear relations among the MZVs, which we call confluent relations. We start with Goncharov's notation for iterated integrals, review some basic notions and properties of iterated integrals, and define a class of relations among iterated integrals, which naturally arise as "solving differential equations step by step". Confluent relations are defined as the limit of these relations when merging two out of the four punctured points. One of the significance of the confluent relations is that it is a rich family and seems to exhaust all the linear relations among the MZVs. As a good reason for this, we show that confluent relations imply the extended double shuffle relations as well as the duality relations. This is a joint work with Nobuo Sato at National Taiwan University.

Hidetaka Kitayama (Wakayama Univ.)
Siegel modular forms with respect to non-split symplectic groups

We denote by G the unitary group of the quaternion hermitian space of rank two over an indefinite quaternion algebra B over the rational number field. Then the group G is a Q-form of $\mathrm{Sp}(2;\mathbb{R})$, and each Q-form of $\mathrm{Sp}(2;\mathbb{R})$ can be obtained in this way. In this talk, we will consider Siegel modular forms for discrete subgroups of $\mathrm{Sp}(2;\mathbb{R})$ which are defined from G in the case where B is division.

Yingkun Li (Technical Univ. Darmstadt)
Average values of higher Green's functions and their factorizations.

By the classical theory of complex multiplication, the Klein j-invariant takes algebraic values at CM points. In their seminal work on singular moduli, Gross and Zagier gave a factorization of the difference between two such values, which can be viewed as the exponential of a special value of a Green's function on the upper half plane. From numerical computations, they conjectured that the special values of certain higher Green's functions also enjoy similar algebraicity property. We will revisit this conjecture and discuss some recent progress.

Steffen Löbrich (Univ. of Cologne)
Niebur-Poincaré Series and Regularized Inner Products

Zagier introduced weight $2k$ cusp forms $f_{k,D}$ associated to quadratic forms of positive discriminant D. We determine the Fourier coefficients of analogues of these functions of weight 2, higher level, and negative discriminant and relate them to traces of singular moduli of Niebur-Poincaré series. This allows us to compute regularized inner products of these functions, which in the higher weight case have been related to evaluations of higher Green's functions at CM-points.

Kenji Makiyama (Kyoto Sangyo Univ.)
A p-adic family of Saito-Kurokawa lifts for a Coleman family and the Bloch-Kato conjecture

We will construct a p-adic family of Saito-Kurokawa lifts for a Coleman family and extend the result of Agarwal and Brown on the Bloch-Kato conjecture for elliptic modular forms of low weights to higher weights. More precisely, we will prove that the p-valuation of the order of the Selmer group of a Coleman deformation is bounded below by the p-valuation of the algebraic part of the critical L-value attached to the initial Hecke eigenform of a Coleman family satisfying some reasonable assumptions given by Agarwal and Brown.

Toshiki Matsusaka (Kyushu Univ.)
Arithmetic formulas for the coefficients of the McKay-Thompson series

By the work of R. Borcherds we know the Fourier coefficients of the elliptic modular j-function are closely related to the Monster group. As a new perspective, M. Kaneko gave an arithmetic formula for the Fourier coefficients expressed in terms of the traces of the CM values of the j-function in 1996. In this talk, we consider analogues of this formula for the McKay-Thompson series. Our main result is an explicit formula for the coefficients expressed in terms of the CM values of certain hauptmoduln. Furthermore, we give some applications.

Takashi Nakamura (Tokyo Univ. of Science)
Selberg's orthonormality conjecture and joint universality of L-functions

In this talk, we introduce the new approach how to use an orthonormality relation of coefficients of Dirichlet series defining given L-functions from the Selberg class to prove joint universality (joint work with Yoonbok Lee, Lukasz Pánkowski).

Hiro-aki Narita (Kumamoto Univ.)
Explicit constructions of non-tempered cusp forms on orthogonal groups of low split ranks

The aim of this talk is to report a recent research on explicit constructions of cusp forms on orthogonal groups of split rank one or two by some lifts from cusp forms on the complex upper half plane. We also discuss cuspidal representations generated by them in terms of the explicit determination of their local components. As for the representation theoretic treatment, the point is to use Sugano's non-archimedean local theory of "Jacobi form formulation" of Oda-Rallis-Schiffman lifting to orthogonal groups of rank two. Sugano's local theory turns out to be useful also for the case of rank one. Such argument leads to non-temperedness of the non-archimedean local components and the explicit determination of the standard L-functions. It should be remarked that the cusp forms taken up in this talk are counterexamples to the Ramanujan conjecture and those in rank one case are real analytic but non-holomorphic. This talk includes a recent joint work with Ameya Pitale for the case of the rank one.

Tomomi Ozawa (Univ. Paris 13)
Eisenstein and CM congruence modules defined over a real quadratic field

Measuring congruences among modular forms over arithmetic rings has good applications to number theory. In particular, Hida has shown in 2013 that the non-existence of the following two types of congruences is almost equivalent to the vanishing of the μ-invariants of the Kubota-Leopoldt p-adic L-function and the Katz anti-cyclotomic p-adic L-function: (1) a congruence mod p between a p-adic family of Eisenstein series and a non-CM cuspidal Hida family; (2) a congruence mod p between a non-CM and a CM cuspidal Hida family. In this talk, I will explain my attempt to describe congruence modules that classify such types of congruences, in the case where the Hilbert modular forms are defined over a real quadratic field of narrow ideal class number one.

Robert Pollack (Boston Univ./MPIM), ABKLS talk
Computing weight 1 forms – a p-adic approach

The computation of Hecke-eigenforms of weight at least 2 is readily accomplished through the theory of modular symbols as these Hecke-eigensystems occur in the cohomology of modular curves. However, the same is not true for weight 1 modular forms which makes computing the dimensions of such spaces difficult let alone the actual system of Hecke-eigenvalues. Recently effective methods for computing such spaces have been introduced building on an algorithm of Kevin Buzzard. In this talk, we present a different, p-adic approach towards computing these spaces which yields upper bounds on both their dimension and on the systems of Hecke-eigenvalues which they can contain.

Nicole Raulf (Univ. Lille), ABKLS talk
On a mean value result for a product of L-functions

The asymptotic behaviour of moments of L-functions is of special interest to number theorists and there are conjectures that predict the shape of the moments for families of L-functions of a given symmetry type. However, only some results for the first few moments are known. In this talk we will consider the asymptotic behaviour of the first moment of the product of a Hecke L-function and a symmetric square L-function. This is joint work with O. Balkanova, G. Bhowmik, D. Frolenkov.

Hiroshi Sakata (Waseda Univ. Senior High School)
A certain level-index changing operator on Jacobi cusp new forms

Let N be an odd square-free integer. We give a Hecke isomorphism map from the space of Jacobi cusp new forms of level 1 and index N satisfying a certain condition into the space of Jacobi cusp new forms of level N, index 1 and the character after reviewing about some known properties of level-index changing operators.

Nobuo Sato (NCTS of National Taiwan Univ.)
On Charlton's conjecture about the multiple zeta values

In this talk, we give a proof of a special case of the generalized cyclic insertion conjecture on the MZVs, which was formulated by Steven Charlton in his thesis. The conjecture is stated in terms of the block notation for MZVs introduced by himself. Charlton's conjecture is a broad generalization of several long unproven families of identities such as Borwein-Bradley-Broadhurst-Lisoněk's cyclic insertion conjecture and certain conjectural identities posed by Hoffman. Our proof is based on certain identities among iterated integrals on a punctured projective line which we found by a search with the aid of computers. This is a joint work with Minoru Hirose at Kyushu University.

Ren-he Su (Kyoto Univ.)
On linear relations between L-values and arithmetic functions
In 1975 Cohen constructed a series of modular forms of half-integral weights. Its q-coefficients contain special values of Dirichlet functions and were used by Cohen to create various equations of them with arithmetic functions. The modular forms are called Cohen-Eisenstein series and were later generalized to the case for Hilbert modular forms. Making use of the generalized forms one can also write down linear equations for the special values of Dirichlet L-functions with respect to certain general real number fields, even in terms of arithmetic functions on rational integers. In this talk I would like to introduce how this works. The idea was originally inspired by Ikeda.

Markus Schwagenscheidt (Technical Univ. Darmstadt)
Regularized theta lifts of integral weight harmonic Maass forms II

In these two talks we will give an overview on developments in the theory of regularized theta lifts of harmonic Maass forms of integral weight. In particular, we report on recent work which extends the Shintani theta lift to harmonic Maass forms. This yields interesting number theoretic applications.

Shingo Sugiyama (Kyushu Univ.)
Explicit trace formulas for symmetric square L-functions for $\mathrm{GL}(2)$ and some applications

Zagier exhibited an indentity between meromorphic functions as a generalization of the Eichler-Selberg trace formula for elliptic modular forms. Later, Jacquet and Zagier de- scribed such a kind of trace formula with complex parameter for adelic GL(2), containing abstractness of its spectral and geometric terms. In this talk, we give a generalization of Zagier's formula to the case of holomorphic Hilbert modular forms in a new way. As applications, we discuss an equidistribution result for Hecke eigenvalues of Hilbert modu- lar forms in the level aspect, and the abundant existence of Hilbert modular forms whose symmetric square L-functions are non-vanishing at points in the critical strip. This is a joint work with Masao Tsuzuki (Sophia University).

Sho Takemori (MPIM)
Construction of vector valued Siegel modular forms and examples of congruences

I will talk about construction vector valued Siegel modular forms (of any vector valued weight) by theta series with pluri-harmonic polynomials. And I will show examples congruences concerning Hecke eigenforms of degree three which are conjectural lift conjectured by Bergstroem, Faber and van der Geer. This talk is based on a joint work with Professor Ibukiyama. If time permits, I would like to talk about congruences concerning with other lifts also.

Shun'ichi Yokoyama (Kyushu Univ.)
Number theory with Magma: for performant and flexible computation

Magma (distributed at the U. Sydney) is a software designed for computations in algebra, number theory, and arithmetic geometry. In this talk, we will introduce Magma and extensive projects with packages we created (or collaborated) with short demos.

Annalena Wernz (RWTH Aachen)
The Hermitian modular group and the orthogonal group

The Hermitian modular group of degree n over an imaginary quadratic field $K = \mathbb{Q}(\sqrt{-m})$ was introduced by Hel Braun in the 1940s as an analogue for the well known Siegel modular group. It acts on the Hermitian half space and the accociated Hermitian modular forms have been studied thoroughly in the past. However, this talk does not concentrate on the modular forms but on the modular group itself. For $n = 2$ and $m \neq 1, 3$, m squarefree, we will prove that the Hermitian modular group $U(2, 2; \mathcal{O}_K)$, where $\mathcal{O}_K$ is the ring of integers in K, is isomorphic to the discriminant kernel of the orthogonal group O(2,4) and we will provide an explicit homomorphism. Furthermore, we compute the normalizer of the Hermitian modular group in the symplectic group and show that it is isomorphic to the integral orthogonal group which is the normalizer of the discriminant kernel in the real orthogonal group.