Designs, Codes, Graphs and Related Areas

Dates: Monday 1st July to Wednesday 3rd July 2013.
Venue: RIMS, Kyoto University, Room 111
Kyoto 606-8502 Japan

Program and Abstracts

1st July

13 : 00 − 13 : 50 Oleg Musin (Univ. of Texas at Brownsville and Yaroslavl State University) 1
Optimal packings of congruent circles on spheres and flat tori

14 : 00 − 14 : 50 Alexey Glazyrin (Univ. of Texas at Brownsville and Yaroslavl State University) 1
The price of SDP relaxations for spherical codes

15 : 10 − 16 : 00 Keisuke Shiromoto (Kumamoto University) 2
On critical exponents of matroids and linear codes

16 : 10 − 17 : 00 Michiaki Onodera (Kyushu University) 2
Evolution equations for quadrature identities

2nd July

9 : 00 − 9 : 50 Ryoh Fuji-Hara (University of Tsukuba) 4
Descendent sets and codes

10 : 00 − 10 : 50 Yuichiro Fujiwara (California Institute of Technology) 5
Codes and designs for quantum error correction

11 : 00 − 11 : 50 Ken-ichi Kawarabayashi (National Institute of Informatics) 5
Combinatorial coloring of 3-colorable graphs

Lunch break

14 : 00 − 14 : 50 Yuan Xu (University of Oregon) 6
Cubature rules and orthogonal polynomial

15 : 00 − 15 : 50 Bruce Reed (McGill University) 6
The structure of a typical H-free graph

16 : 00 − 16 : 50 Satoshi Aoki (Kagoshima University) 6
Markov chain Monte Carlo methods for regular two-level fractional factorial designs and cut ideals

Banquet (18:30–)

3rd July

9 : 00 − 9 : 50 Alexander Barg (University of Maryland) 7
On the theory of association schemes

10 : 00 − 10 : 50 Hiroto Sekido (Kyoto University) 7
An approximate approach to E-optimal designs for weighted polynomial regression by using Tchebycheff systems and orthogonal polynomials

11 : 00 − 11 : 50 Mikio Kano (Ibaraki University) 7
Discrete geometry on 3 colored point sets in the plane

Lunch break

14 : 00 − 14 : 50 Ferenc Szöllősi (Tohoku University) 10
Equiangular lines with angle 1/5 and Seidel matrices with 3 distinct eigenvalues

15 : 00 − 15 : 50 Akihiro Higashitani (Osaka University) 10
Ehrhart polynomials of polytopes and orthogonal polynomial systems

16 : 00 − 16 : 50 Takayuki Okuda (Tohoku University) 10
Relation among designs on compact homogeneous spaces

Optimal packings of congruent circles on spheres and flat tori

Oleg Musin

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We consider packings of congruent N circles on spheres (the Tammes problem) and flat square tori. Toroidal packings are interesting due to a practical reason - the problem of super resolution of images. We classified all locally optimal spherical arrangements up to $N = 11$. For packings on tori we have found optimal arrangements for $N = 6, 7$ and 8. Surprisingly, for the case $N = 7$ there are three different optimal arrangements. Our proofs are based on computer enumerations of spherical and toroidal irreducible contact graphs. This is joint work with Alexey Tarasov (spheres) and Anton Nikitenko (tori).

The price of SDP relaxations for spherical codes

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A *spherical code* $\mathcal{C} = \{x_1, \dots, x_M\} \subset S^{n-1}$ is a subset of points on the sphere in $\mathbb{R}^n$. Another way to define a spherical code is through a symmetric $M \times M$ matrix T with $t_{ii} = 1$ $(1 \le i \le M)$, $-1 \le t_{ij} \le 1$ $(1 \le i \ne j \le M)$ that satisfies

$$T \succeq 0; \quad \operatorname{rank}(T) \le n. \tag{1}$$

In other words, T is a Gram matrix of a set of unit vectors that form $\mathcal{C}$.

The Delsarte method and its SDP extension rely on a set of relaxations of the condition $\operatorname{rank}(T) \le n$ for a configuration of points on the sphere. Let $G_k^{(n)}(t)$ be the classical Gegenbauer polynomial of degree k, and consider the $M \times M$ matrix $(G_k^{(n)}(t_{ij}))_{1 \le i,j \le M}$ where the matrix elements are the values of $G_k^{(n)}$ evaluated at $t_{ij} = (x_i, x_j)$. By a well-known *Schoenberg's theorem*, this matrix is positive semidefinite for all k

$$(G_k^{(n)}(t_{ij})) \succeq 0 \quad (k = 1, 2, \dots). \tag{2}$$

The Delsarte method further replaces (2) with the conditions

$$\sum_{i,j} G_k^{(n)}(t_{ij}) \ge 0 \quad (k = 1, 2, \dots). \tag{3}$$

This talk is devoted to the question of the gap between the exact description of codes (1) and the relaxed ones (2)-(3).

This is joint work with Oleg Musin.

On critical exponents of matroids and linear codes

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The critical exponent of a matroid is one of the important parameters in matroid theory which is related to the critical problem. A representable matroid over a finite field is corresponding to a linear code over the field. In this talk, we give a bound on critical exponents of linear codes and give a construction of linear codes which attain the equality of the bound.

Evolution equations for quadrature identities

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One of the classical problems in potential theory is to specify a surface Γ for a prescribed electric charge density μ in such a way that the uniform distribution of electric charges on Γ produces the same potential (at least in a neighborhood of the infinity) as μ. To derive a mathematical formulation of the problem precisely, let E be the fundamental solution of $-\Delta$ in $\mathbb{R}^N$, i.e.,

$$E(x) := \begin{cases} -\dfrac{1}{2\pi}\log|x| & (N=2), \\ \dfrac{1}{N(N-2)\omega_N|x|^{N-2}} & (N \geq 3), \end{cases}$$

where ω_N is the area of the unit ball in $\mathbb{R}^N$, and let $\mathcal{H}^{N-1}\lfloor\Gamma$ denote the $(N-1)$-dimensional Hausdorff measure restricted to Γ. Then, the problem can be stated as follows: for a prescribed finite Radon measure μ with compact support in $\mathbb{R}^N$, find a $(N-1)$-dimensional closed surface Γ enclosing a bounded domain Ω such that $E * \mu = E * \mathcal{H}^{N-1}\lfloor\Gamma$ in $\mathbb{R}^N \setminus \overline{\Omega}$, i.e.,

$$\int E(x-y)\,d\mu(y) = \int_\Gamma E(x-y)\,d\mathcal{H}^{N-1}(y) \quad \left(x \in \mathbb{R}^N \setminus \overline{\Omega}\right). \tag{4}$$

It can be shown that (4) is equivalent to the identity

$$\int h\,d\mu = \int_\Gamma h\,d\mathcal{H}^{N-1} \tag{5}$$

holding for all harmonic functions h defined in a neighborhood of $\overline{\Omega}$.

Definition 1. *A closed surface Γ satisfying* (5) *is called a quadrature surface of μ for harmonic functions.*

The mean value property of harmonic functions implies that (5) is valid when $\mu = N\omega_N\delta_0$ and $\Gamma = \partial B(0,1)$, where δ_0 is the Dirac measure supported at the origin and $B(0,1)$ is the unit ball in $\mathbb{R}^N$. Thus, the identity (5) can be seen as a generalization of the mean value formula for harmonic functions.

The existence of a quadrature surface Γ of a prescribed μ has been studied by several authors with different approaches. Developing the idea of super/subsolutions of Beurling [2], Henrot [4] was able to prove that the existence of Γ is guaranteed when a supersolution and a subsolution are available. Gustafsson & Shahgholian [3] followed a variational approach developed by Alt & Caffarelli [1], namely, they consider the minimization problem for the functional

$$J(u) := \int_{\mathbb{R}^N} \left(|\nabla u|^2 - 2fu + \chi_{\{u>0\}}\right)\,dx,$$

and obtain the existence and regularity of a minimizer u. Then, u satisfies the Euler-Lagrange equation

$$-\Delta u = f\lfloor\Omega - \mathcal{H}^{N-1}\lfloor\partial\Omega, \qquad \Omega = \{u > 0\},$$

and the existence of such a u implies that $\Gamma = \partial\Omega$ is a quadrature surface of μ with $d\mu = f\,dx$.

However, as pointed out by Henrot [4], the uniqueness of a quadrature surface cannot hold in general. The collapse of uniqueness seems to indicate a bifurcation phenomenon of solutions to (5) with a parametrized measure $\mu = \mu(t)$. Hence, toward understanding of the uniqueness issue, we need to consider the corresponding family of surfaces $\Gamma = \Gamma(t)$. In this respect, it is natural to ask if there is an evolution equation describing the moving surfaces $\{\Gamma(t)\}_{t>0}$ such that each $\Gamma(t)$ is a quadrature surface of a given parametrized measure $\mu(t)$. As a matter of fact, when $\mu(t) = t\delta_0 + \chi_{\Omega(0)}$ and the identity (5) is replaced by

$$\int h\,d\mu = \int_\Omega h\,dx, \tag{6}$$

it is known that the Hele-Shaw flow, a model of interface dynamics in fluid mechanics, surprisingly, plays the desired role. Here, analogously, a domain Ω satisfying (6) is called a quadrature domain of μ. Hence, the investigation of the evolution of quadrature domains is reduced to that of the Hele-Shaw flow, and the latter has been successfully proceeded by complex analysis and the theory of partial differential equations.

We introduce the following geometric evolution equation:

$$\begin{aligned} &v_n = p \quad \text{for } x \in \partial\Omega(t), \\ &\text{where} \begin{cases} -\Delta p = \mu & \text{for } x \in \Omega(t), \\ (N-1)Hp + \dfrac{\partial p}{\partial n} = 0 & \text{for } x \in \partial\Omega(t), \end{cases} \end{aligned} \tag{7}$$

where v_n is the growing speed of $\partial\Omega(t)$ in the outer normal direction and H is the mean curvature of $\partial\Omega(t)$. The following theorem shows that, as desired, for a given $\partial\Omega(0)$ as initial surface, the solution to (7) turns out to be a one-parameter family of quadrature surfaces. Moreover, we will see that (7) is the only possible flow having this property. Here, we call $\{\partial\Omega(t)\}_{0\le t<T}$ a $C^{3+\alpha}$ family of surfaces if each $\partial\Omega(t)$ is of $C^{3+\alpha}$ and its time derivative is of $C^{2+\alpha}$, namely, $\partial\Omega(t)$ can be locally represented as a graph of a function in the Hölder space $C^{3+\alpha}$ and its time derivative is in $C^{2+\alpha}$.

Theorem 2. *Let $\{\partial\Omega(t)\}_{0\le t<T}$ be a $C^{3+\alpha}$ family of surfaces, and assume that each $\partial\Omega(t)$ has positive mean curvature. Then, each $\partial\Omega(t)$ is a quadrature surface of $\mu(t) := t\mu + \mathcal{H}^{N-1}\lfloor\partial\Omega(0)$ if and only if $\{\partial\Omega(t)\}_{0\le t<T}$ is a solution to* (7).

At this point, we are led to a fundamental question: Does the equation (7) really possess a unique smooth solution? The following theorem affirmatively answers this question. Here, $\{\partial\Omega(t)\}_{0\le t<T}$ is called a $h^{3+\alpha}$ solution if it is a $h^{3+\alpha}$ family of surfaces and satisfies (7), where $h^{3+\alpha}$ is the so-called little Hölder space and is defined as the closure of the Schwartz space $\mathcal{S}$ of rapidly decreasing functions in the topology of the Hölder space $C^{3+\alpha}$.

Theorem 3. *There exists a unique $h^{3+\alpha}$ solution $\{\partial\Omega(t)\}_{0\le t<T}$ to* (7) *for any $h^{3+\alpha}$ initial surface $\partial\Omega(0)$ with positive mean curvature.*

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Descendent sets and codes

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Let $S = \{1, 2, ..., q\}$ and $C \subset S^n$. To each $i = 1, \ldots, q$, we associate the set $C(i)$ of the elements appearing in the i-th coordinate, meaning,

$$C(i) = \{c_i \mid (c_1, c_2, ..., c_n) \in C\}.$$

The descendent set of C, $desc(C)$, is the set of all possible n-tuples of S^n such that the elements at the i-th coordinate of $desc(C)$ are from $C(i)$:

$$desc(C) = C(1) \times C(2) \times \cdots \times C(n).$$

The n-tuples of C are called *parents*.

Several different codes are defined by descendent sets. We here call them the *descendent codes.* There is a descendent code which is defined by a simple condition of descendent set. Let $\mathfrak{C}$ be a set of n-tuples of S^n satisfying $desc(C) \neq desc(D)$ for any $C, D \subset \mathfrak{C}$ such that $C \neq D$ and $|C|, |D| \leq t$. $\mathfrak{C}$ is called a *t-separable code.*

Theorem 1 (M.Cheng and Y. Miao 2011)**.** *When $t = 2$,*

$$M \leq q^{n-1} + q(q-1)/2,$$

where M is the number of code words of $\mathfrak{C}$.

Since the descendent set is a set of n-tuples, there are not many convenient tools for manipulating sets. Therefore, there are some constructions for only $n = 2$ and $t = 2, 3$.

Here we represent a descendent set as a vector over the finite field of order 2. Let e_i be the i-th identity vector of length q, meaning, the vector of length q whose i-th coordinate is 1 and the others are all 0, and $E_q = \{e_1, e_2, ..., e_q\}$. Consider a map σ defined as follow:

$$\sigma : s \in S \mapsto e_s \in E_q$$

Then, for any $\mathbf{x} = (c_1.c_2, ..., c_n) \in S^n$

$$\sigma(\mathbf{x}) = (\sigma(c_1), \sigma(c_2), ..., \sigma(c_n)) \in {E_q}^n$$

When $C_0 = \{\mathbf{x}, \mathbf{y}\} \subset S^n$ we define a vector which corresponds to the descendent set of $\{\mathbf{x}, \mathbf{y}\}$:

$$\begin{aligned} dv(\mathbf{x}, \mathbf{y}) &= \sigma(\mathbf{x}) * \sigma(\mathbf{y}) + \{\sigma(\mathbf{x}) + \sigma(\mathbf{y})\} \\ &= \sigma(\mathbf{x}) \vee \sigma(\mathbf{y}) \end{aligned}$$

where $*$ is the bit-wise multiplication over F_2.
We call $dv(\mathbf{x}, \mathbf{y})$ the *descendent vector* of $\{\mathbf{x}, \mathbf{y}\}$.

For a set containing more than two elements, we use the following property: Let $C_0 \subset S^n, \mathbf{x} \in S^n \setminus C_0$,

$$dv(C_0 \cup \{\mathbf{x}\}) = dv(C_0) * \sigma(\mathbf{x}) + \{dv(C_0) + \sigma(\mathbf{x})\}$$

The descendent vector is useful for defining, analyzing, and constructing those codes. Here the basic properties between descendent sets and descendent vectors:

(1) For any $C_0, C_1 \subset S^n$, $desc(C_0) = desc(C_1)$ if and only if $dv(C_0) = dv(C_1)$.

(2) For any $C_0, C_1 \subset S^n$, $desc(C_0) \cap desc(C_1) = \emptyset$ if and only if there exists the zero element of F_2^q in the vector $dv(C_0) * dv(C_1)$.

(3) For any $\mathbf{x} \in S^n$ and $C_0 \subseteq S^n$, $\mathbf{x}$ is an element of $desc(C_0)$ if and only if there exists no zero of F_2^q in the vector $dv(C_0) * \sigma(\mathbf{x})$ (or $\sigma(\mathbf{x}) = dv(C_0) * \sigma(\mathbf{x})$).

(4) For any $\mathbf{x} \in S^n$ and $C_0 \subseteq S^n$, $\mathbf{x} \in desc(C_0)$ if and only if $dv(C_0) = dv(C_0 \cup \{\mathbf{x}\})$.

(5) For any $C_0, C_1 \subseteq S^n$, $C_0 \neq C_1$, $desc(C_0) \subseteq desc(C_1)$ if and only if $dv(C_0) * dv(C_1) = dv(C_0)$.

(6) For any $C_0, C_1 \subseteq S^n, dv(C_0 \cap C_1) \preceq dv(C_0) * dv(C_1)$, where $X \preceq Y$ if and only if $X = X * Y$.

(7) For any $C_0, C_1 \subseteq S^n, dv(C_0 \cup C_1) = dv(C_0) * dv(C_1) + \{dv(C_0) + dv(C_1)\}$.

We discuss the descendent codes focused on the $t-$separable codes, and their properties and constructions.

Codes and designs for quantum error correction

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Suppressing the effect of decoherence plays a vital role in the theory of quantum information processing. Despite many physicists' skepticisms about its feasibility, mathematics proved in 1997 the existence of an error correction scheme in the quantum domain, giving the birth to the field of quantum error correction. Since then, the new field has seen remarkably rapid progress in various directions including experimental realizations of quantum error-correcting codes. While quantum information science draws on many branches of physics, computer science, and mathematics, combinatorics is rapidly becoming an indispensable mathematical tool to the study of quantum error correction. In this talk, we overview the latest developments in quantum error correction where combinatorial design theory and coding theory play the central role.

Combinatorial coloring of 3-colorable graphs

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Recognizing 3-colorable graph is one of the most famous NP-complete problems [Garey, Johnson, and Stockmeyer 1974]. The problem of coloring 3-colorable graphs in polynomial time with as few colors as possible has been intensively studied: $O(n^{1/2})$ colors [Wigderson J. ACM, 1982], $O(n^{2/5})$ colors [Blum 1989], $O(n^{3/8})$ colors [Blum J. ACM, 1990], $O(n^{1/4})$ colors [Karger, Motwani, Sudan, J. ACM, 1994], $O(n^{3/14}) = O(n^{0.2142})$ colors [Blum and Karger IPL'97], $O(n^{0.2111})$ colors [Arora, Chlamtac, and Charikar STOC'06], and $O(n^{0.2072})$ [Chlamtac FOCS'07]. Recently together with M, Thorup, we got down to $O(n^{0.2049})$ colors [FOCS'12]. In this talk we get down further to $O(n^{0.19996}) = o(n^{1/5})$ colors.

Since 1994, the best bounds have all been obtained balancing between combinatorial and semi-definite approaches. We present a new combinatorial recursion that only makes sense in collaboration with semi-definite programming. We specifically target the worst-case for semi-definite programming: high degrees. By focusing on the interplay, we obtained the biggest improvement in the exponent since 1997.

Cubature rules and orthogonal polynomials

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Let Π_n^d denote the space of polynomials of degree n in d real variables. Let W be a weight function defined on a domain $\Omega \subset \mathbb{R}^d$. A cubature rule of degree $2n-1$ is a finite sum of function evaluations that approximates the integral against W,

$$\int_\Omega f(x)W(x)dx = \sum_{k=1}^{N} \lambda_k f(x_k), \qquad \forall f \in \Pi_{2n-1}^d,$$

where $\lambda_k \in \mathbb{R}$ and $x_k \in \mathbb{R}^d$. We are interested in cubature rules that have minimal or near minimal N for a fixed n. The nodes $\{x_k\}$ of such a cubature rule are often common zeros of certain orthogonal polynomials of degree n with respect to W.

Unlike the case of one variable ($d = 1$), where minimal quadrature rules always have zeros of orthogonal polynomials as their nodes, the relation between cubature rules and zeros of orthogonal polynomials of several variables is much more involved and less clear, as it is essentially a relation between algebraic ideals and their varieties. This talk is aimed at explaining problems in this direction and what are known.

The structure of a typical H-free graph

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We show that the vertex set of almost every graph which does not contain a cycle of length six as an induced subgraph can be partitioned into a stable set and a subgraph containing neither a stable set of size three nor an induced matching of size 2. We discuss similar precise characterizations of typical graphs without induced cycles of length k for all k. We present a conjecture about the structure of almost every graph without H as an induced subgraph for arbitrary H. We discuss various pieces of evidence in support of this conjecture.

Markov chain Monte Carlo methods for regular two-level fractional factorial designs and cut ideals

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It is known that a set of binomial generators of cut ideals I_G of a graph G corresponds to a Markov basis of the binary graph model of the suspension of G. In this talk, we give another application of cut ideals to statistics. We show that a set of binomial generators of cut ideals corresponds to a Markov basis of some regular two-level fractional factorial design. As application, we give a Markov basis of degree 2 for designs defined by at most two relations.

On the theory of association schemes

Alexander Barg

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We discuss a way of defining association schemes on infinite, possibly uncountable sets. Joint work with Maxim Skriganov (Steklov Institute, St. Petersburg, Russia).

An approximate approach to E-optimal designs for weighted polynomial regression by using Tchebycheff systems and orthogonal polynomials

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E-optimal designs are defined as the multisets of experimental conditions which minimize the maximum axis of the confidence ellipsoid of estimators. Only a few E-optimal designs are calculated exactly and analytically, and some of them are calculated by using Tchebycheff systems. In this talk, a new method for constructing E-optimal designs approximately for weighted polynomial regression with a nonnegative weight function is proposed. Notions of the Tchebycheff systems and orthogonal polynomials are used in the proposed method.

Discrete geometry on 3 colored point sets in the plane

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1 3 colored point sets in the plane

Let R, B and G denote disjoint sets of red points, blue points and green points in the plane, respectively. If no three points of $R \cup B \cup G$ are collinear, we say that R, B and G are *in general position* in the plane. We always assume that given points are in general position.

We begin with the following well-known theorem on two colored point sets in the plane. Note that a *geometric graph* is a graph drawn in the plane whose edges are straight line segments, and every edge of an *alternating matching* joins two points with distinct colors.

Theorem 1 ([3]). *If $|R| = |B|$, then there exists an alternating non-crossing geometric perfect matching on $R \cup B$ (see Figure 1).*

We generalize the above theorem by considering 3 colored point sets. The proof of the following theorem is basically similar to that of the above Theorem 1, but more difficult.

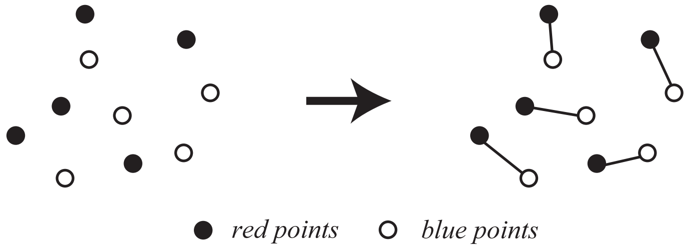

Figure 1: An alternating non-crossing geometric perfect matching on $R \cup B$.

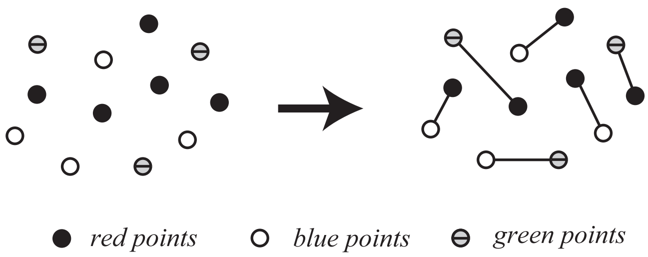

Figure 2: An alternating non-crossing geometric perfect matching on $R \cup B \cup G$.

Corollary 2 (Kano, Suzuki, Uno [4]). *If $|R \cup B \cup G| = 2n$, $|R| \le n$, $|B| \le n$ and $|G| \le n$, then there exists an alternating non-crossing geometric perfect matching on $R \cup B \cup G$.*

It is known as the discrete version of Ham-Sandwich theorem that if $|R| = 2m$ and $|B| = 2n$, then there exists a bisector line l such that $|left(l) \cap R| = m$ and $|left(l) \cap B| = n$. It is easy to see that there exist configurations of 3 colored points in the plane such that there exists no bisector line for three colors. For a set X of points in the plane, we denote the *convex hull* of X by $conv(X)$.

Theorem 3 (Bereg and Kano [2]). *Assume that $|R| = |B| = |G| = n$ for $n \ge 2$. If all the vertices of $conv(R \cup B \cup G)$ are red, then there exists a line l such that $|right(l) \cap R| = |right(l) \cap B| = |right(l) \cap G| = k$ for some integer $1 \le k \le n-1$ (see Figure 3).*

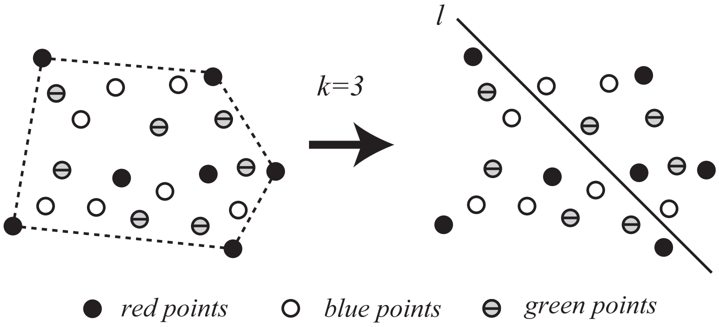

Figure 3: All the vertices of $conv(R \cup B \cup G)$ are red; An line l such that $right(l)$ contains exactly $k = 3$ red points, k blue points and k green points.

We explain a sketch of the proof of Theorem 3.

Theorem 4 (Berege etc. [1]). *Assume that n red points and n blue points and n green points lie on a circle in the plane. The for every integer $1 \le k \le n-1$, there exist two intervals I and J on the circle such that $I \cup J$ contains exactly k red points, k blue ponts and k green points (see Figure 4).*

We shall explain a sketch of the proof of the above Theorem 4.

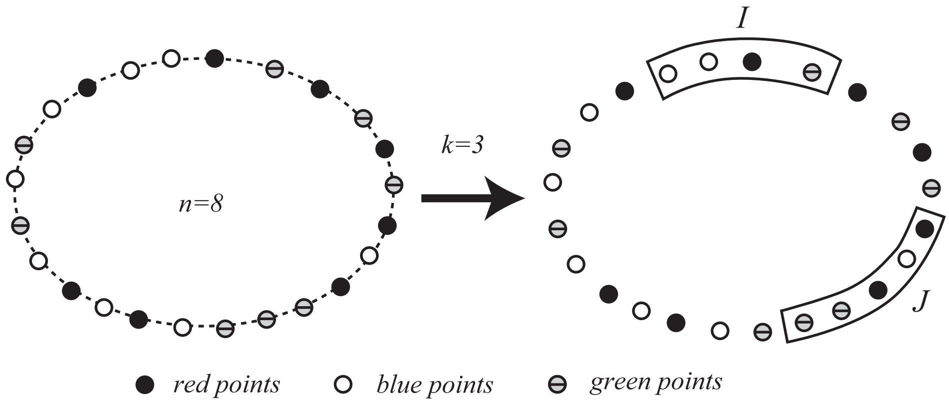

Figure 4: n red points, n blue points and n green points are given on a cricle in the plane; An interval $I \cup J$ contains exactly k red points, k blue points and k green points.

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Equiangular lines with angle $1/5$ and Seidel matrices with 3 distinct eigenvalues

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(joint work in progress with Gary Greaves)

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Let $k, d > 1$. A set of lines, represented by the unit vectors $v_1, v_2, \ldots, v_k \in \mathbb{R}^d$ is called equiangular, if there exists a constant $c \in \mathbb{R}$ such that $|\langle v_i, v_j\rangle| = c$ for all $1 \leq i < j \leq k$. The Seidel matrix of the system is the $\{0, \pm 1\}$ matrix S, $S_{ij} = (\langle v_i, v_j\rangle - \delta_{ij})/c$. It is easy to see that $-1/c$ is the smallest eigenvalues of S with multiplicity of at least $k - d$. In this talk we give an overview of Seidel matrices with smallest eigenvalue -5 and explore Seidel matrices with 3 distinct eigenvalues.

Ehrhart polynomials of polytopes and orthogonal polynomial systems

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Ehrhart polynomials of integral convex polytopes are one of well-studied objects in enumerative combinatorics and to study roots of Ehrhart polynomials is a fundamental and essential work in this area.

On the other hand, orthogonal polynomial systems (OPS) are of importance and appear in several areas of mathematics. It is well known that an OPS has an outstanding property on its zeros if the corresponding moment functional is positive-definite.

In this talk, we will focus on the Ehrhart polynomials of reflexive polytopes, which have a remarkable property, and discuss the relation between them and OPS. There are some interesting examples of reflexive polytopes whose Ehrhart polynomials are OPS's with respect to positive-definite moment functionals, which will be presented in this talk.

Relation among designs on compact homogeneous spaces

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The concept of spherical design was introduced by Delsarte–Goethals–Seidel in 1977 and similar definitions were known for designs on rank one compact symmetric spaces, real or complex Grassmannian manifolds (by Bachoc–Nebe (2002) and Roy (2010), respectively), unitary groups (by Roy (2009)), flag manifolds (by Meyer (2009)) and complex spheres (by Roy–Suda (preprint)). In this talk, we generalize such definitions for designs on a general compact homogeneous space G/K and prove that for a compact Lie group G and closed subgroups K and L of G such that K is included in L, we can construct a design on G/K as a "product of a design on G/L and that on L/K". As an application of our result, we also give a construction of designs on a real Grassmannian manifold $G^R_{m,n}$ from a sequence of spherical designs on $S^1, S^2, ..., S^{n-1}$, where $G^R_{m,n}$ denotes the manifold consisting of m-dimensional subspaces of an n-dimensional real vector space.

Poster exhibition

Shoko Chisaki (Tokyo University of Science)
Difference System of Sets from Lines in $PG(4,2)$

Hirotake Kurihara (Kitakyushu National College of Technology)
Relations between great antipodal sets and designs with the smallest cardinalities on the complex Grassmannian manifolds and the unitary groups

Sho Suda（Aichi University of Education）
Weighing matrices and related combinatorial objects

Banquet Information

Banquet will be held from 18:30 on July 2nd, at Ganko Sanjō-honten (がんこ三条本店), which is located within 2 minutes walking distance of Sanjō-eki.

Banquet charge: Faculty 6,500 JPY; Student 5,000JPY

Ganko Sanjō-honten
URL: www.gankofood.co.jp/shop/detail/wa-sanjo/

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