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ELLIPSOIDAL T-DESIGN

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ABSTRACT. A *spherical t-design* is a finite set X of points on the unit sphere in $\mathbb{R}^n$ which satisfies

$$\frac{1}{|X|} \sum_{x \in X} P(x) = \frac{1}{\text{Vol}(S^{n-1})} \int_{S^{n-1}} P(x) d\sigma(x)$$

for all polynomials $P(x)$ with degree $\leq t$. The right-hand side is the surface integral over S^{n-1}. Recently, Miezaki generalized this concept to arbitrary (potentially infinite) subsets T of positive integers, as opposed to the finite set of degrees $\leq t$. In the case where $n = 2$, he used the theory of complex multiplication by $\mathbb{Z}[i]$ to show that there are infinitely many spherical T-designs (namely lattice points of fixed norm) for the set $T = \mathbb{Z}^+ \setminus 4\mathbb{Z}$. We generalize the concept of spherical T-designs to ellipses in the case of imaginary quadratic orders with class number 1. This work relies on the theory of modular forms with complex multiplication. I will also talk about my recent work with Wei-Lun which extends the work to any classnumber.

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