ELLIPSOIDAL T-DESIGN

BADRI VISHAL PANDEY

Abstract. A spherical t-design is a finite set $X$ of points on the unit sphere in $\mathbb{R}^n$ which satisfies

$$
\frac{1}{|X|} \sum_{x \in X} P(x) = \frac{1}{\text{Vol}(S^{n-1})} \int_{S^{n-1}} P(x) d\sigma(x)
$$

for all polynomials $P(x)$ with degree $\leq t$. The right-hand side is the surface integral over $S^{n-1}$. Recently, Miezaki generalized this concept to arbitrary (potentially infinite) subsets $T$ of positive integers, as opposed to the finite set of degrees $\leq t$. In the case where $n = 2$, he used the theory of complex multiplication by $\mathbb{Z}[i]$ to show that there are infinitely many spherical $T$-designs (namely lattice points of fixed norm) for the set $T = \mathbb{Z}^+ \setminus 4\mathbb{Z}$. We generalize the concept of spherical $T$-designs to ellipses in the case of imaginary quadratic orders with class number 1. This work relies on the theory of modular forms with complex multiplication. I will also talk about my recent work with Wei-Lun which extends the work to any classnumber.